Stressing correlations and volatilities: A consistent modeling approach

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In recent years dependencies between financial assets have been notoriously high. Whole markets increasingly behave like one asset: as the Financial Times puts it, markets are either going up like a rocket or fall down like a stone. Therefore diversification works worse than anticipated, and the risk of a financial meltdown increases. For regulating banks it is essential to take the increase in asset dependencies into account. The Basel Committee on Banking Supervision (BCBS) has suggested that banks compute a value-at-risk under stress, that is, when dependencies are increased [cf. BCBS 2009b, BCBS 2011b]. However, the committee acknowledges that banks that wish to fulfill this requirement face difficult technical questions [cf. BCBS 2011a]. We develop a new approach to the definition of stress scenarios for asset dependencies. The stress scenarios are based on historical experience and correspond to pre-specified levels of stress. Furthermore, they fulfill the requirements in BCBS [cf. BCBS 2009b; BCBS 2011b]. We compare correlations in a normal market and under stress and explore consequences for value-at-risk. The results based on our model confirm estimates of the Basel Committee.

We suggest a new asset price model where dependencies, measured by asset return correlations, are determined by a common market factor which describes the state of the market. This dependence on a common market state is the key to stressing correlations and volatilities in a consistent and intuitive way. In our model the vector of volatilities $\sigma$ and the correlation matrix $\rho$ depend on one and the same market state $F$. The market state $F$ is generic; to comply with suggestions of the Basel Committee we define the market state as the realized drift of a market index over a rolling time window.

This market factor captures market-wide movements in equity-prices. We estimate the dependence structure of volatilities $\sigma(F)$ and correlations $\rho(F)$ on the market state $F$ from daily stock prices. Figure 1 shows estimates for selected pairs of stocks. Negative values for the market state indicate market downturns, a market state around zero or slightly positive indicates a normal market, strongly positive market states indicate a bull market. We observe that correlations are increased in bear markets (that is, market state $F \leq -0.5$), and stable in normal and bull markets ($F \geq -0.5$). The horizontal lines in Figure 1 are estimated constant correlations and volatilities in a corresponding model that assumes only constant volatilities and correlations. The difference between market state dependent volatilities as well as correlations and their constant counterparts, respectively, indicates how

Figure 1: Typical dependency structures of correlation $\rho$ and volatilities $\sigma_1, \sigma_2$ on the market state

(a) Colgate – Exxon

(b) Walt Disney – Pfizer

(c) Walmart – Johnson & Johnson

Data from 1990-2010; Annualised realised drift of the S&P 500 over a rolling window of the past 75 business days.
strongly these quantities change in a crisis and can be interpreted as a measure for financial contagion. The market state is computed as a realized drift over a fixed number of past daily observations. This number of past observations has a convenient interpretation as the memory of the market. Our estimates yield that for all assets considered the optimal memory is about 75 business days.

Stressing portfolio correlation cannot be done in a bivariate manner because this may result in an invalid correlation matrix. Therefore, for a portfolio consisting of \( n \) stocks we simultaneously estimate the vector of volatilities \( \sigma(F) \) and the \( n \times n \) correlation matrix \( \rho(F) \). As a result, the vector of volatilities and the correlation matrix are known functions \( \sigma(F), \rho(F) \) of the market state \( F \). We propose to define risk scenarios that comply with the requirements of the BCBS [see BCBS 2009b] by shifting the market state to predefined stress levels that are based on the time series of historical realizations of our market state \( F \). More precisely, we shift the market state \( F \) to a level \( f \) with, for example, \( \alpha \in \{0.1\%, 1\%, 5\%\} \), such that the percentage of historically observed market states are below \( f \);

\[
\rho(f) = (p_{ij}(f))_{i,j = 1,\ldots,n},
\]

and the vector of stressed volatilities

\[
\sigma(f) = (\sigma_1(f), \ldots, \sigma_n(f)).
\]

Since the market state is defined as the realized drift of an appropriate index, it captures the systematic market risk component of the stock portfolio. By design the proposed stress scenarios for volatilities and correlations fulfill the minimum requirements posed by the BCBS [see BCBS 2011b]. In particular, the choice of the stress level relates to the severity of the stress scenario. Moreover, volatilities and correlations are stressed in a consistent way because they are stressed simultaneously and based on one and the same market factor \( F \).

**VaR analysis under stress**

To illustrate the idea we analyze a portfolio’s value-at-risk under stress. Let us assume that we invest an amount \( V_0 \) in some sample portfolio. For asset weights \( \gamma \) we compute a 10-days ‘stressed value-at-risk’ for level \( \alpha \) by

\[
\text{VaR}^{\text{stressed}}_{\alpha} = -V_0 \left( \exp \left( \sqrt{\frac{10}{250}} \Phi^{-1}(\alpha) \sum_{i,j=1}^{n} \gamma_i \gamma_j \sigma_i(f)a_{ij}(f) \rho_{ij}(f) \right) - 1 \right),
\]

where volatilities and correlations are evaluated at the stress level \( f \) of historically observed market states \( F \). The function \( \Phi \) is the cumulative distribution function of the standard normal distribution. Furthermore, we compute a ‘nonstressed value-at-risk’ for level \( \alpha \) by

\[
\text{VaR}^{\text{nonstressed}}_{\alpha} = -V_0 \left( \exp \left( \sqrt{\frac{10}{250}} \Phi^{-1}(\alpha) \sum_{i,j=1}^{n} \gamma_i \gamma_j \sigma^\text{const}_i \sigma^\text{const}_j \rho_{ij}^\text{const} \right) - 1 \right)
\]

for a sample portfolio of \( 20 \) stocks where we assume equal asset weights \( \gamma \). Figure 2b shows the ratio of stressed and non-stressed value-at-risk,

\[
\alpha \rightarrow \frac{\text{VaR}^{\text{stressed}}_{\alpha}}{\text{VaR}^{\text{nonstressed}}_{\alpha}}, \quad \alpha \in \{0.01, 0.05, 0.1\}
\]

We observe that the stressed 1% value-at-risk \( \text{VaR}^{\text{stressed}}_{0.01} \) computed with market state dependent volatilities and correlations is 4 times higher than the value-at-risk \( \text{VaR}^{\text{stressed}}_{0.01} \) computed with constant volatilities and correlations. Our results for the stressed value-at-risk at the 1%-level confirm the findings of BCBS [see BCBS 2009a] who report that the ratio of the stressed value-at-risk and the non-stressed value-at-risk as computed by banks is in the range of 0.68 – 7 with median 2.6.

**Conclusions**

To summarize the advantages of our approach we conclude that, firstly, by defining stress scenarios for volatilities and correlations by (1)-(2), we fulfill the minimum requirements posed by the BCBS [see BCBS 2009b], that is, “there should be a risk factor that is designed to capture market-wide movements in equity-prices (e.g. a market index)”. Moreover, volatilities and correlations are stressed in a consistent way because they are stressed simultaneously and based on one and the same market factor \( F \). Secondly, our approach confirms

The model is estimated on Jan 2004-Nov 2010 for a portfolio of 20 stocks and portfolio value of 100$. 

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**Figure 2: Estimated ten-day VaR for different stress levels \( \alpha \)**

(a) Value-at-Risk

(b) Ratio of stressed to non-stressed value-at-risk
findings of the impact study BCBS [2009a] on how much a stressed value-at-risk should exceed a standard value-at-risk. Thirdly, the correlation matrix (1) and the vector of volatilities (2) can be used as inputs for a market risk analysis in any model where daily returns are assumed to be normally distributed.

Becker and Schmidt [see Becker/Schmidt 2011] also find that the model can outperform standard approaches like the Dynamic Conditional Correlation GARCH model by Engle [see Engle 2002] in capturing the dynamics of correlations and volatilities within given samples. In forthcoming research they investigate alternative drivers for correlations and the model’s capabilities to measure systemic risk.

Reference literature


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